For Candidates Admitted From 2015-2017

15 MMA 42C REG.NO................

M.Sc., DEGREE EXAMINATIONS, APRIL 2019

MATHEMATICS SEMESTER : IV

FLUID DYNAMICS

Time : 3 HRS. Max.Marks: 75

PART - A ( 10 X 2 =20)

ANSWER ALL THE QUESTIONS

1. Define : Stream line
2. Write down the anyone Navier Stokes equation.
3. Write down the Euler's equation.
4. Write down the Bernoulli's equations for steady motion.
5. Define : Stream function for two dimensional motion.
6. Write down the Laplace's equation in Cartesian coordinates
7. Write down the Reynold's law of dynamical similarity.
8. Define Coutte flow for Flat plates.
9. Write down the Prandtl's boundary layer equations.
10. Write down the Von Karman integral relation of the boundary layer.

PART –B ( 5 X 5 =25)

ANSWER ALL THE QUESTIONS

11. a. The velocity vector in flow field is given q = i( Az-By) + j( Bx – Cz) + k(Cy – Ax) where A, B and C are non zero constants Determine the equations of the vertex lines.

(or)

b. Consider an incompressible steady flow with constants viscosity, The velocity components are given by

$$w(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(\frac{-dp}{dx}\right)\frac{y}{h}(1-\frac{y}{h})$$

u=w=0 If the body force is neglected does u(y) satisfy the equation of motion h,U and dp/dx are constants and p = p(x).

12. a. Calculate the velocity of the water jet as for $p_2 = 0.1MN/m^2$ $p_1 = 0.2MN/m^2$, $A_2/A_1 = 0.01$ *and* $h = 5m$

(or)

b. Explain Stokes' theorem.

13. a. Show that the velocity vector is everywhere tangent to lines in the x-y plane along which $\psi(x,y) =$ constant.

(or)

b. Verify that the stream function $\psi$ and velocity potential $\phi$ of a three dimensional doublet flow satisfy the Laplace equation.

14. a. Water at $20^0$C flows between two large parallel plates at a distance of 1.5mm apart. If the average velocity is 0.15 m/s find i) the wall shearing stress ii) the frictional coefficient.

(or)

b. Discuss the flow between two coaxial cylinders.

15. a. Explain boundary layer on a surface with pressure gradient.

(or)

b. Derive the Von Karman integral relation of the boundary layer.

PART –C ( 3 X 10 =30)

ANSWER ANY THREE QUESTIONS

16. Derive the energy equation.
17. Derive the Momentum theorem.
18. Explain three dimensional axially symmetric flow for i) Uniform flow ii) Radial flow – source or sink.
19. a) Determine the maximum value of the velocity profile in the annular space between two coaxial cylinders.

b) If $r_1$ = 50mm, $r_2$= 75mm and the volumetric flow of water Q = 0.006 $m^3$/s, Calculate i) the pressure drop ii) The maximum value of $\theta_z$ ii) The shearing stress at the wall of both cylinders.

20. Explain the boundary layer along a Flat plate for the Blasius solution.

(OR)

(b) If $m$ represents a probability distribution on $X$, then prove that the measure of dissonance in evidence as defined by $E(m) = -\sum_{A \in \mathcal{F}} m(A) \log_2 Pl(A)$ becomes equivalent to the shannon entropy.

15. (a) Explain fuzzy similarity relation.

(OR)

(b) Illustrate the fuzzy model of decision making.

## PART – C (3 × 10 = 30)

## ANSWER ANY THREE QUESTIONS.

16. Show that fuzzy set operations of unions, intersection, and continuous complement that satisfy the law of excluded middle and the law of contradiction are not idempotent or distributive.

17. Explain transitive max-min closure for a relation. Obtain this for the fuzzy relation R(X, X) given by

| R | A | B | C | D |
|---|---|---|---|---|
| A | .7 | .5 | 0 | 0 |
| B | 0 | 0 | 0 | 1 |
| C | 0 | .4 | 0 | 0 |
| D | 0 | 0 | .8 | 0 |

18. Prove that a belief measure Bel on a finite power set $\mathcal{P}(X)$ is probability measure if and only its basic assignment m is given by $m(\{x\}) = Bel(\{x\})$ and $m(A) = 0$ for all subsets of $X$ that are not singletons.

19. Construct and discuss in detail the Boltzmann entropy.

20. Briefly explain fuzzy set theory in the field of computer science.

For Candidates Admitted From 2015-2017

15 MMA 43C ROLL NO …………..

# M.Sc., DEGREE EXAMINATIONS, APRIL 2019

## SEMESTER – IV MATHEMATICS

## FUZZY LOGIC AND FUZZY SETS

TIME: 3 Hrs Max. Marks: 75

### PART – A (10 × 2 = 20)

ANSWER ALL QUESTIONS

1. Define equality of two fuzzy sets.
2. Define L-fuzzy sets
3. Define anti-symmetric fuzzy binary relation.
4. Define tolerance relations.
5. Write the continuity axioms of fuzzy measures.
6. Define a plausibility measure
7. Define index of fuzziness.
8. Write the Shannon entropy in terms of probability theory.
9. Write some application in the field of fuzzy set theory.
10. How fuzzy set theory is used in the field of interpersonal communication.

### PART – B (5 × 5 = 25)

ANSWER ALL QUESTIONS.

11. (a) Prove that every fuzzy complement has at most one equilibrium.

(OR)

(b) Prove that for all $a, b \in [0,1]$, $i(a,b) \geq i_{min}(a,b)$.

12. (a) Explain in detail: Fuzzy equivalence.

(OR)

(b) Write the several types of ordering relation in fuzzy.

13. (a) With usual notations prove that $Pl(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B \in \mathcal{P}(X)$.

(OR)

(b) Discuss with examples: noninteractive consonant bodies of evidence.

14. (a) Prove that $H(X|Y) = H(X,Y) - H(Y)$.

P.T.O

simple functions $\phi$ and $\psi$ it is necessary and sufficient that f be measurable.

18. Let f be an increasing real valued function on the interval [a,b]. Prove that f is differentiable almost everywhere Prove that also that f' measurable and $\int_a^b f'(x)dx \le f(b)-f(a)$.

19. State and prove Minkowski inequality.

20. State and prove the Riez representation theorem.

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(or)

b) If Q is odd and Q > 0, then prove that $\left(\frac{-1}{Q}\right) = (-1)^{(Q-1)/2}$.

15. a) Find the number of integers in the set $\rho = \{1, 2, 3, \ldots, 6300\}$ that are divisible by neither 3 nor 4, also the number divisible by none of 3, 4 or 5.

(or)

b) Find all integers x and y such that $147x + 258y = 369$.

## SECTION – C (3 x 10 = 30)

**Answer any THREE Questions:-**

16. Find $g = (b, c)$ where $b = 5033464705$ and $c = 3137640337$, and determine x and y such that $bx + cy = g$.

17. Determine whether the system $x \equiv 3 \pmod{10}$, $x \equiv 8 \pmod{15}$, $x \equiv 5 \pmod{84}$ has a solution, and find them all, if any exist.

18. If p is a prime then prove that there exist $\phi(\phi(p^2)) = (p-1)\,\phi(p-1)$ primitive roots modulo $p^2$.

19. State and prove Gauss lemma.

20. Find all solutions of $999x - 49y = 5000$.

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**For Candidates Admitted From 2018**

# M.Sc., DEGREE EXAMINATIONS, APRIL - 2019

## II SEMESTER, MATHEMATICS

## 18MMA25E – NUMBER THEORY

**Time : 3 Hrs** **Max. Marks: 75**

**SECTION – A (10 x 2 = 20)**

**Answer ALL Questions:-**

1. State Euclid theorem.
2. Define composite number with an example.
3. State Fermat's theorem.
4. Write Wolstenholme's congruence.
5. Define Primitive root.
6. What is $n^{th}$ power residue?
7. Define quadratic residue.
8. What is form?
9. State the Polignac's formula.
10. Write down the Mobius inversion formula.

**SECTION – B (5 x 5 = 25)**

**Answer ALL Questions:-**

11\. a) State and prove the division Algorithm.

**(or)**

b) By using the Euclidean algorithm, find the greatest common divisor of 42823 and 6409.

12\. a) Find the least positive integer x such that $x \equiv 5 \pmod 7$, $x \equiv 7 \pmod{11}$, and $x \equiv 3 \pmod{13}$.

**(or)**

b) State and prove Wilson's theorem.

13\. a) State and prove Euler's Criterion.

**(or)**

b) Determine the number of solutions of the congruence $x^4 \equiv 61 \pmod{117}$.

14\. a) If p and q are distinct odd primes, then prove that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\{(p-1)/2\}\{(q-1)/2\}}$

P.T.O

$$\text{Maximize } z = x_1^2 + 3x_2^2 + 5x_3^2$$
$$\text{subject to } \quad x_1 + x_2 + 3x_3 = 2;$$
$$5x_1 + 2x_2 + x_3 = 5$$

**(or)**

b. Find the solution of the following problem.

$$\text{minimize } f(x, y) = Kx^{-1}y^{-2}$$
$$\text{subject to the conditions } g(x, y) = x^2 + y^2 - a^2 = 0$$

15. a. Explain the Wolfe's modified Simplex method.

**(or)**

b. Use the Kuhn Tucker conditions to solve the following non linear programming problem

$$\text{maximize } z = 2x_1 + 12x_1x_2 - 7x_2^2$$
$$\text{subject to the constraints } 2x_1 + 5x_2 \le 98$$

## PART –C ( 3 X 10 =30)
## ANSWER ANY THREE QUESTIONS

16. Solve the following parametric linear programming problem.

$$\text{maximize } z = (\lambda - 1)x_1 + x_2$$
$$\text{subject to the constraints}$$
$$x_1 + 2x_2 \le 10,\ 2x_1 + x_2 \le 11,\ x_1 - 2x_2 \le 3;\ x_1, x_2 \ge 0$$

17. Customers arrive at a milk booth for the required service. Assume that the inter arrival and service times are constant and given by 1.8 and 4 time units respectively. Simulate the system by hand computations for 14 time units. What is the average waiting time per customer? What is the percentage idle time of the facility? [Assume that the system starts at t=0].

18. Solve the following game graphically

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix} \end{array}$$

19. Solve :

$$\text{maximize } \quad z = 2x_1 - x_1^2 + x_2$$
$$\text{subject to the constraints}$$
$$2x_1 + 3x_2 \le 6,\quad 2x_1 + x_2 \le 4 \text{ and } x_1, x_2 \ge 0$$

20. Solve by Beale's method

$$\text{maximize } f(x) = 2x_1 + 3x_2 - 2x_2^2 \text{ subject to the constraints}$$
$$x_1 + 4x_2 \le 4,\ x_1 + x_2 \le 2,\ x_1, x_2 \ge 0$$

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For Candidates Admitted From 2018

18 MMA 23C REG.NO..........

M.Sc. DEGREE EXAMINATIONS, APRIL 2019

MATHEMATICS SEMESTER : II

OPERATIONS RESEARCH

Time : 3 HRS. Max.Marks: 75

PART –A ( 10 X 2 =20)

ANSWER ALL THE QUESTIONS

1. What are the two requirements must be met to start the LP optimal and infeasible?
2. Write a note on parametric changes in C.
3. What is comparison matrix?
4. What is the difference between making a decision under risk and under uncertainity?
5. Define the value of game.
6. What is two person zero sum game?
7. What is Non linear programming problem?
8. Write a note on constrained optimization.
9. Write the general form of quadratic programming problem.
10. What is Kuhn- Tucker condition?

PART –B ( 5 X 5 =25)

ANSWER ALL THE QUESTIONS

11. a. Write the revised simplex algorithm.

(or)

b. Solve :

$$Minimize\ z = 3x_1 + 2x_2 + x_3$$
$$subject\ to\ 3x_1 + x_2 + x_3 \geq 3,\ -3x_1 + 3x_2 + x_3 \geq 6$$
$$x_1 + x_2 + x_3 \leq 3;\ x_1, x_2, x_3 \geq 0$$

12. a. Given the cummulative density function f(x) of the random variable x, $-\infty < x < \infty$, Prove that the random variable $z = f(x)$, $0 \leq z \leq 1$, has the following uniform 0- identity function $f(z) = 1$, $0 \leq z \leq 1$.

(or)

b. A small industry finds from the past data that the cost of making an item is Rs. 25 the selling price of an item is Rs. 30 if it sold within a week and it could be disposed of at Rs. 20 per item at the end of the week.

| Weekly sales : | < 3 | 4 | 5 | 6 | 7 | >8 |
|---|---|---|---|---|---|---|
| No. of weeks : | 0 | 10 | 20 | 40 | 30 | 0 |

Find the optimum number of items per week the industry should produce.

13. a. Use Dynamic programming to find the value of

$$\max imize\ z = y_1, y_2, y_3$$
$$subject\ to\ the\ constraints\ : y_1 + y_2 + y_3 = 5$$
$$y_1, y_2, y_3 \geq 0$$

(or)

b. Solve the following game and determine the value of the game.

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \end{array}$$

14.a. Obtain the set of necessary conditions for the non linear programming problem.

P.T.O

OR

B) Prove the inclusion relations

i) $Quasinormal \subseteq subnormal$

ii) $Normaloid \subseteq Spectraloid$

15. A) State and prove Young inequality.

OR

B) Show that every invertible p-hyponormal operator is a log-hyponormal operator.

PART – C

Answer THREE Questions (3×10 = 30)

16. If $T$ is an operator on a Hilbert space $H$ over the complex scalars $C$, then prove the following

(i) $T$ is normal if and only if $||Tx|| = ||T^*x||$ for all $x \in H$. .

(ii) $T$ is self-adjoint if and only if $(Tx, x)$ is real for all $x \in H$.

(iii) $T$ is unitary if and only if $||Tx|| = ||T^*x|| = ||x||$ for all $x \in H$.

17. Let $T = U|T|$ be the polar decomposition of an operator $T$. Then prove that

(1) If $T$ is binormal, then so is $U$.

(2) If $T$ is quasinormal, then so is $U$; $U$ = isometry $\oplus 0$ on $N(T)^{\perp} \oplus N(T)$.

(3) 1f $T$ is normal, then so is $U$; $U$ = isometry $\oplus 0$ on $N(T)^{\perp} \oplus N(T)$.

18. State and prove Toeplitz-Hausdorff theorem.

19. Let $T$ be a hyponormal operator. Then prove that the following hold.

i) $T - \mu$ is hyponormal for any $\mu \in C$

ii) $T$ is a transaloid operator.

iii) $T^{-1}$ is hyponormal if $T^{-1}$ exists.

20. State and prove generalized Furuta inequality.

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For Candidates Admitted From 2015-2017

15MMA4HC

M.Sc. DEGREE EXAMINATION, APRIL 2019
SEMESTER IV – MATHEMATICS
OPERATOR THEORY

Time: 3Hrs Max. Marks: 75

PART – A

Answer ALL Questions (10×2 = 20)

1. Define a bounded operator.
2. If $P$ is a projection operator then show that $||x||^2 = ||Px||^2 + ||(I-P)x||^2$.
3. Define isometry operator.
4. Let $T = UP$ be the polar decomposition of an operator $T$. Then show that $T$ is normal if and only if $U$ commutes with $P$.
5. If $T \geq cI$ for some $c > 0$, then show that $T$ is invertible.
6. Define spectrum of an operator.
7. Define paranormal operator.
8. Show that every normal operator is convexoid.
9. Define p-hyponormal operator.
10. If $A \geq B > 0$, then show that $logA > logB$.

PART – B

Answer ALL Questions (5×5 = 25)

11. A) Prove that for any linear operator $T$ on a Hilbert space $H$, the following statements are mutually equivalent:
    (i) $T$ is bounded.
    (ii) $T$ is continuous on the whole space $H$
    (iii) $T$ is continuous on some point $x_0$ on $H$.

OR

B) State and prove the generalized Schwarz inequality.

12. A) Let $T = U|T|$ be the polar decomposition of an operator $T$ on a Hilbert space $H$. Then show that $T^* = U^*|T^*|$ is also the polar decomposition of an operator $T^*$.

OR

B) State and prove Fuglede-Putnam theorem.

13. A) If an operator $T$ is self-adjoint, then show that $\sigma(T)$ is a subset of the real line.

OR

B) If T is an operator such that $||I - T|| < 1$, then show that $T$ is invertible.

14. A) If $T$ is a paranormal operator then prove that the following hold.
    i) $T$ is a normaloid operator.
    ii) If $T$ is an invertible paranormal operator, then prove that $T^{-1}$ is also an invertible paranormal operator.

P.T.O

18. Let u be harmonic in a region $\Re$ . Also let p (x,y,z) be a given point in $\Re$ and s(p,r) be a sphere with centre at p such that S(P,r) is completely contained in the domain of harmonicity of u. Then prove that

$$u(p) = \bar{u}(r) = \frac{1}{4\pi r^2} \iint_{s(p,r)} u(q)ds.$$

19. Obtain the solution of diffusion equation in spherical coordinates.
20. State and prove Duhamel's principle for wave equation.

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**For Candidates Admitted From 2018**

**18 MMA 24C** **REG.NO..........**

**M.Sc. DEGREE EXAMINATIONS, APRIL 2019**

**MATHEMATICS** **SEMESTER : II**

**PARTIAL DIFFERENTIAL EQUATIONS**

**Time : 3 HRS.** **Max.Marks: 75**

**PART –A ( 10 X 2 =20)**

**ANSWER ALL THE QUESTIONS**

1. Eliminate the arbitrary constants a and b form the equation
   z =(x+a) (y +b)
2. Define Cauchy's characteristics equations of the differential equation.
3. Write the three canonical forms in PDE.
4. Define adjoint operator.
5. Write Laplace equation in cylindrical co-ordinates.
6. Define the boundary value problem of first kind.
7. Let f(t) be any continuous function. Then prove that

$$\int_{-\infty}^{\infty} \delta(t-a)\, f(t)dt = f(a)$$

8. Write the diffusion equation in cylindrical Co-ordinates.
9. What are the characteristics if $u_{tt} - c^2u_{xx} = 0$ ?
10. Write the D' Alembert's solution of the one dimensional wave equation.

**PART – B ( 5 X 5 =25)**

**ANSWER ALL THE QUESTIONS**

11. a. Find the general Integral of the equation.
    ( x - y)p + (y-x-z)q = z.

    **(or)**

    b. Show that the equation $xp - yq = x$ , $x^2 p + q = xz$ are compatible and find their solution.

12. a. Reduce the equation to a canonical form
    $(1+x^2)u_{xx} + (1-y^2)u_{yy} + xu_x + yu_y = 0$

    **(or)**

    b. Construct an operator adjoint to the Laplace equation
    $d(y)=u_{xx} + u_{yy}$.

13. a. Derive Poisson's equation.

    **(or)**

    b. Find the solution of the Laplace equation by the method of separation of variables.

14. a. Derive the elementary solution of diffusion equation.

    **(or)**

    b. Determine the temperature T(r,t) in the infinite cylinder $0 \leq r \leq a$ when the initial temperature is T(r,0) = f(r) and the surface r = a is maintained at $0^0$ temperature.

15. a. Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions.
    i) u(0, t) = u (2,t) = 0  ii) u (x,0) = $\sin^3 \frac{\pi x}{2}$  iii) $u_t$ (x,0) = 0

    **(or)**

    b. Derive solution of one dimensional wave equation in cylindrical coordinates.

**PART – C ( 3 X 10 =30)**

**ANSWER ANY THREE QUESTIONS**

16. Use Charpit's method to solve the partial differential equation
    $z^2 = pqxy$.
17. Show that the Green's function for the equation

$$\frac{\partial^2 u}{\partial x \partial y} + u = 0, \quad v(x,y,\xi,\eta) = J_0\sqrt{2(x-\xi)(y-\eta)}$$

where $J_0$ denotes Bessel's function of the first kind of order zero.

P.T.O

13. a) Suppose $\lambda$ and $\mu$ are the two independent elastic constants (Lame's coefficients), then show that the Young's modulus E and the Poisson's ratio $v$ are equal to $\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$ and $\frac{\lambda}{2(\lambda+\mu)}$.

(or)

b) Derive the stress- strain relations for isotropic materials.

14. a) A thick-walled steel cylinder with radii a=5 cm and b=10 cm is subjected to an internal pressure p. The yield stress in tension for the material is 350 MPa. Using a factor of safety of 1.5, determine the Maximum normal stress and maximum shear stress, where $E = 207 \times 10^6 kPa, v = 0.25$.

(or)

b) Determine the shape for a disk with uniform stress, i.e. $\sigma_r = \sigma\_\theta$.

15. a) Determine the bending moment due to the circumferential stress across a section of a thin hollow disk subjected to radial thermal variation.

(or)

b) obtain the normal stresses for a solid sphere with radial temperature variation.

## PART – C (3 X 10 = 30)
## ANSWER ANY THREE QUESTIONS

16. Determine the principal stresses and their axes for the states of stress characterised by the flowing stress matrix (Units are 1000 kPa)

$$[\tau_{ij}] = \begin{bmatrix} 18 & 0 & 24 \\ 0 & -50 & 0 \\ 24 & 0 & 32 \end{bmatrix}.$$

17. The displacement field for a body is given by $\boldsymbol{u} = \left((x^2 + y)\boldsymbol{i} + (y + z)\boldsymbol{j} + (x^2 + 2z^2)\boldsymbol{k}\right)10^{-3}$. At a point P(2,2,3), consider two line segments PQ and PR having the direction cosies $PQ: \eta_{x1} = \eta_{y1} = \eta_{z1} = \frac{1}{\sqrt{3}}$ and $PR: \eta_{x2} = \eta_{y2} = \frac{1}{\sqrt{2}}, \eta_{z2} = 0$. Determine the angle between the two segments before and after deformation.

18. A cubical element is subjected to the following state of stress: $\sigma_x = 100MPa$, $\sigma_y = -20\ MPa, \sigma_z = -40\ MPa, \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$. Assuming the material to be homogeneous and isotropic, determine the principal shear strains and the octahedral shear strain, if $E = 2 \times 10^5$ and $v = 0.25$ .

19. A pipe made of steel has a tensile elastic limit $\sigma_y = 275\ MPa$ and $E = 207 \times 10^6 kPa$. If the pipe has an internal radius $a = 5cm$ and is subjected to an internal pressure p = $70 \times 10^3 kPa$, determine the proper thickness for the pie wall according to the major theories of failure. Use a factor of safety $N = \frac{4}{3}$.

20. Derive the equations of normal stresses in straight beams due to thermal loading.

For Candidates Admitted From 2015-2017

2015-17 MMA 44C

ROLL NO .

# M.SC. DEGREE EXAMINATIONS, APRIL 2019

## SEMESTER IV – MATHEMATICS

## ELECTIVE: SOLID MECHANICS

Time: 3 Hrs

Max. Marks: 75

### PART – A (10 X 2 = 20)

### ANSWER ALL QUESTIONS

1. Define stress.
2. Define Principal stress and principal plane.
3. What is Deformation?
4. Suppose $|e_{ij}| = \begin{bmatrix} \epsilon_{xx} - e & e_{xy} & e_{xz} \\ e_{xy} & \epsilon_{yy} - e & e_{yz} \\ e_{xz} & e_{yz} & \epsilon_{zz} - e \end{bmatrix} + \begin{bmatrix} e & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & e \end{bmatrix}$, then find the linear invariant of the strain deviator.
5. Write down the relationship between shear modulus and bulk modulus interms of Young's modulus and Poisson's ration.
6. Write any two components of strain as a linear function of the six components of stress.
7. Define Lame's constant $\lambda$ and $\mu$.
8. Define the boundary conditions of think hollow sphere problems.
9. Define the equilibrium of equations of thermal stresses.
10. Define traction boundary condition.

### PART – B (5 X 5 = 25)

### ANSWER ALL QUESTIONS

11. a) Consider the stress function $\phi(r,\theta)$ with $\sigma_r = \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}, \sigma_\theta = \frac{\partial^2\phi}{\partial r^2}$ and $\tau_{r\theta} = -\frac{1}{r}\frac{\partial^2\phi}{\partial r\partial\theta} + \frac{1}{r^2}\frac{\partial\phi}{\partial\theta}$. Show that in the absence of body forces, these satisfy the differential equation of equilibrium.

(or)

b) The state of stress at a point is characterised by the components $\sigma_x = 100\ MPa$, $\sigma_y = -40\ MPa, \sigma_z = 80\ MPa, \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$. Determine the extremum values of the shear stresses and their associated normal stresses.

12. a) The following displacement field is imposed on a body $\boldsymbol{u} = (xy\,\boldsymbol{i} + 3x^2z\,\boldsymbol{j} + 4\,\boldsymbol{k})10^{-2}$. Consider a point P and a neighbouring point Q where PQ has the direction cosines $\eta_x = 0.2, \eta_y = 0.8$ and $\eta_z = 0.555$. Point P has coordinates (2,1,3). If $PQ = \Delta s$, find the components of P'Q' after deformation.

(or)

b) Derive the differential equations of equilibrium in Cartesian coordinate system.

(b) Let X be a metric space with metric $d$. Define $\overline{d} : X \times X \to \mathbb{R}$ by the equation $\overline{d}(x,y) = \min\{d(x,y), 1\}$. Then prove that $\overline{d}$ is a metric that induces the same topology as $d$

13. (a) Let $A$ be a connected subspace of $X$. If $A \subset B \subset \overline{A}$, then prove that $B$ is also connected

(OR)

(b) Every compact subspace of a Hausdorff space is closed

14. (a) Show that every metrizable space is normal

(OR)

(b) Prove that a product of completely regular spaces is completely regular

15. (a) Let $X$ be a set and let $\mathfrak{D}$ be a collection of subsets of $X$ that is maximal with respect to the finite intersection property. If A is a subset of X that intersects every element of $\mathfrak{D}$ then prove that A is an element of $\mathfrak{D}$.

(OR)

(b) If $X$ is compactly generated then prove that a function $f: X \to Y$ is continuous if for each compact subspace $C$ of $X$, the restricted function $f|C$ is continuous.

## <u>PART–C</u> (3 × 10 = 30)

## ANSWER ANY THREE QUESTIONS.

16. (a) Let $Y$ be a subspace of $X$. Then prove that a set $A$ is closed in $Y$ if and only if it equals the intersection of a closed set of $X$ with $Y$.

(b) Let $Y$ be a subspace of $X$; let $A$ be a subset of $Y$; let $\overline{A}$ denote the closure of $A$ in $X$. Then prove that closure of $A$ in $Y$ equals $\overline{A} \cap Y$.

17. State and prove uniform limit theorem.
18. Prove that the product of finitely many compact spaces is compact.
19. State and prove Urysohn lemma
20. State and prove Ascoli's theorem

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For Candidates Admitted From 2018

SUB.CODE: 18MMA21C

REG. NO …………..

# M.Sc., DEGREE EXAMINATIONS, April-2019

## SEMESTER – II MATHEMATICS

## TOPOLOGY

Time: 3 Hrs

Max. Marks: 75

### PART – A(10×2=20)

ANSWER ALL QUESTIONS

1. Define finite complement topology.
2. Define limit point of a set in a topological space
3. What is topological imbedding?
4. Define standard bounded metric corresponding to a metric d
5. Give an example of a space which is not connected
6. Define linear continuum on a set.
7. Show that subspace of a second countable is second countable
8. Define separable space
9. State Tychonoff theorem
10. Define totally bounded space on a metric space

### PART – B($5 \times 5 = 25$)

ANSWER ALL QUESTIONS.

11. (a) Let $X$ be an ordered set in the order topology; let $Y$ be a subset of $X$ that is convex in $X$. Then prove that the order topology on $Y$ is same as the topology $Y$ inherits as a subspace of $X$.

(OR)

(b) Define order topology. If $\mathfrak{B}$ is a basis for the topology of $X$ and $\mathcal{C}$ is a basis for the topology of $Y$, then prove that the collection $\mathcal{D} = \{B \times C : B \in \mathfrak{B} \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$

12. (a) State and prove pasting lemma

(OR)